Problem 5196. Determine the last six digits of the product $(2010)\left(5^{2014}\right)$

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Clearly the last digit of $N = (2010)\left(5^{2014}\right)$is 0. Therefore in order to find the last six digits of N it is enough to calculate the last five digits of $(201)\left(5^{2014}\right)$.

Let us first calculate a few power of 5, and to do it we need to know just the last five digits of the previous power of 5:

$$\begin{array}{llll} 5^1 = 5 & 5^2 = 25 & 5^3 = 25 & 5^4 = 625 \\ 5^5 = 3125 & 5^6 = 15625 & 5^7 = 78125 & 5^8 = \cdots 90625 \\ 5^9 = \cdots 53125 & 5^{10} = \cdots 65625 & 5^{11} = \cdots 28125 & 5^{12} = \cdots 40625 \\ 5^{13} = \cdots 03125 & 5^{14} = \cdots 15625 & & \end{array}$$

Observe that the last five digit of 5^{14} are the same as those of 5^6. Therefore, starting with 5^6 the last five digits of powers of 5 will repeat periodically:

$$15625, 78125, 90625, 53125, 65625, 28125, 40625, 0325, 15625, \ldots$$

and so on. This means that increasing the exponent of eight does not change the last five digits of powers of 5. Since $2014 = 6 + 8 \cdot 251$, it follows that 5^6 and 5^{2014} have the same last five digits, so

$$201 \cdot 5^{2014} \equiv 201 \cdot 5^6 \equiv 201 \cdot 15625 \equiv 40625 \pmod{10^5}$$

and this implies that the last six digits of $(2010)\left(5^{2014}\right)$ are 406250. □